ABSTRACTS

Darbouxian First Integrals and Invariants for Real Quadratic Systems having an Invariant Conic

Laurent Cairó **Université d'Orléans, France**
Jaume Llibre **Universitat Autònoma de Barcelona, Spain**

Abstract: We apply the Darboux theory to study the integrability of real quadratic differential systems having an invariant conic.The fact that two intersecting straight lines or two parallel straight lines are particular cases of conics allows us to study simultaneously the integrability of quadratic systems having at least two invariant straight lines real or complex.

Topological Equivalence of Vector Fields in Higher Dimension

Felipe Cano **Universidad de Valladolid, Spain**

Abstract: Not notified.

Period Function for a Class of Hamiltonian Systems

Anna Cima, Armengol Gasull ,Francesc Mañosas **Universitat Autònoma de Barcelona, Spain**

Abstract: In this talk we deal with the period function of the class of Hamiltonian systems $dot x=-H_y, dot y=H_x$ where $H(x,y)$ has the special form $H(x,y)=F(x)+G(y)$ and the origin is a non degenerated center. More concretely, if $T(h)$ denotes the period of the periodic orbit contained in $H(x,y)=h$ we solve the inverse problem of characterizing all systems with a given function $T(h)$. We also characterize the limiting behaviour of T at infinity when the origin is a global center and apply this result to prove, among other results, that there are no polynomial isochronous centers in this family.

Interval Exchange and Piecewise Linear Maps

Milton Cobo **Universidad de Campinas, Brazil**

Abstract: We will study the differenciablity of conjugations between interval exchange maps and bijective piecewise linear maps of the interval, which we will call affine interval exchanges. If T is a (uniquely ergodic) interval exchange and F is an affine interval exchange which is conjugated to T, then we will show that the differenciability of the conjugation depend only on the vector of derivatives of the map F, and that this conjugation is not absolutely continuous in most of the cases, implying that F has an invariant measure which is singular with respect to Lebesgue.

An approach to the Center-Focus Problem via Pseudo Normal Forms

Amadeu Delshams, Antoni Guillamon, J. Tomás Lázaro **Universitat Politècnica de Catalunya, Spain**

Abstract: The existence of a convergent transformation $z=Phi(zeta,theta)$ leading a general system $dot{z}=F(z,theta)$, $zin {bf R}^2$, $thetain {bf T}$, in a neighborhood of a hyperbolic fixed point, into its Birkhoff Normal Form, is a classical result due to J. Moser (1956). It is also known its convergence for autonomous systems of the plane around elliptic fixed points. One of the applications of these results is to the problem of determining whether a nonlinear planar vector field with center-type linear part has a center or not ({it Center–Focus Problem}). Our approach, which is also due to D. DeLatte and J. Moser, is based on looking for a close to the identity vector field Phi and two vector fields B and N (of an special form) satisfying the relation $DPhi cdot N + B = Fcirc Phi$. It is proved their convergence for any kind of vector fields on the plane (autonomous or not) in the {it hyperbolic} case and for autonomous ones in the {it elliptic} case. Note that, in particular, if $Bequiv 0$, the vector field Phi constitutes a change of variables and, therefore, we get a Birkhoff Normal Form. That is, our system is integrable. In the elliptic case, this implies that the equilibrium is a center. The coefficients of B can be thought as obstructions to have centers and so play a similar role to the {it Lyapunov constants}, as well as the coefficients of N contain the {it constants of period}. Finally, we want to stress that all this process is carried out constructively and has been implemented on a computer.

About the Convergence of Kuratowski in Differential Equations

Zofia Denkowska **Université d'Angers, France**

Abstract: This is a convergence of closed sets, extensively used in PDE, close to the De Giorgi convergence, useful to obtain some results in generalized PDE (cf. my common work with my husband, Z. Denkowski, Birkahauser). I have just come across some applications of this convergence in analytic geometry (convergence of intersections of analytic sets, complex case) and am trying to make it out for the first return map. The link seems unknown to me.

Perturbations from an Elliptic Hamiltonian of Degree Four

Freddy Dumortier **Limburgs Universitair Centrum, Belgium**

Abstract: The talk deals with Li'enard equations of the form $x'= y, y'= P(x) + y Q(x)$, with P and Q polynomials of degree respectively 3 and 2. We present a number of results obtained in cooperation with Li Chengzhi. Attention goes to perturbations of the Hamiltonian vector fields with an elliptic Hamiltonian of degree four, treating the different cases: saddle loop, two-saddle cycle, cuspidal loop, global center and figure-eight loop. The study permits to prove the existence of Li'enard equations of type (3,2) with a quadruple limit cycle occuring in a complete swallowtail bifurcation of limit cycles. It also permits to prove the occurence of Li'enard equations of type (3,2) having five limit cycles.

Global Estimates on the Number of Limit Cycles for Li'enard's Equations

Jean Pierre Françoise **Université P.M. Curie, Paris VI, France**

Abstract: New complex analytic methods yield explicit estimates for the number of limit cycles of a Li'enard equation.

First Derivative of the Period Function of a Centre

Emilio Freire **Universidad de Sevilla, Spain**

Abstract: Given a centre of a planar differential system, we extend the use of the Lie bracket to the determination of the monotonicity character of the period function. As far as we know, there are no general methods to study this function, and the use of commutators and Lie brackets has been restricted to prove isochronicity. A new criterion to find isochronous centres, without looking either for commutators or for linearizations, is given.

On Limit Cycles for Quadratic Systems with Invariant Algebraic Curves

Javier Chavarriga **Universitat de Lleida, Spain**
Isaac A. García **Universitat de Lleida, Spain**

Abstract: In this paper we consider real quadratic systems. We present new criteria for the existence and uniqueness of limit cycles for such systems by using Darbouxian particular solutions. Some results are based on the study of such systems in ${CP^2}$. We also generalize the well-know result of Bautin on the nonexistence of limit cycles for quadratic Lotka-Volterra systems.

Upper Bounds for the Number of Limit Cycles of Some Liénard Differential Equations

Armengol Gasull **Universitat Autònoma de Barcelona, Spain**
Hector Giacomini **Université de Tours, France**

Abstract: It is well known that the Van der Pol differential equation has at most one limit cycle. There is a recent proof of Cherkas of this fact by using a clever Dulac function. In this talk we extend this idea to give the exact number of limit cycles of other examples of Li'enard differential equations. This talk is based on a joint work with Hector Giacomini.

The Infinitesimal 16th Hilbert Problem in the Quadratic Case

Lubomir Gavrilov **Université Paul Sabatier, France**

Abstract: Let $H(x,y)$ be a real cubic polynomial with four {em distinct} critical values (in a complex domain) and let $ X_H= H_y frac{partial}{partial x} – H_x frac{partial}{partial y} $ be the corresponding Hamiltonian vector field. We show that there is a neighborhood ${cal U}$ of X_H in the space of all quadratic plane vector fields, such that any $Xin {cal U}$ has at most two limit cycles.

Injectivity of $cal{C}^1$ Maps $R^2 to R^2$ at Infinity and Planar Vector Fields

Carlos Gutiérrez **Universidade de Sao Paulo, Brazil**

Abstract: Let $X:{R}^2setminus B_r to {R}^2$ be a $cal{C}^1$ map, where $r>0$ and $B_r = { pin {R}^2 : vertvert p vertvert le r}.$ (a) If for some $epsilon >0$ and for all $pin {R}^2setminus B_r,$ no eigenvalue of $DX(p)$ belongs to $(-epsilon, infty),$ then there exists $sge r$, such that $X,$ restricted to ${R}^2setminus B_r$ is injective; (b) If for some $epsilon >0$ and for all $pin {R}^2setminus B_r,$ no eigenvalue of $DX(p)$ belongs to $(-epsilon, 0)cup { zin{C}: {R}e(z)ge 0 },$ then the point $infty$, of the Riemann sphere ${R}^2 cup {infty}, $is either an attractor or a repellor of $x'= X(x)$.

Isochronicity in a Family of Planar Polynomial Hamiltonian Systems

Xavier Jarqué **Universitat Autònoma de Barcelona, Spain**
Jordi Villadelprat **Universitat Autònoma de Barcelona, Spain**

Abstract: The talk deals with centers of polynomial Hamiltonian systems and we are interested in the isochronous ones. We prove that every center of a polynomial Hamiltonian system of degree four (that is, with its homogeneous part of degree four not identically zero) is non-isochronous. The proof uses the geometric properties of the period annulus and it requires the study of the Hamiltonian systems associated to a Hamiltonian of the form $H(x,y)=A(x)+B(x)y+C(x)y^2+D(x)y^3$.

Monodromy, Stability, and Bifurcation of a Limit Cycle from Degenerate Singular Points of Certain Planar Vector Fields

Víctor Mañosa **Universitat Politècnica de Catalunya, Spain**

Abstract: We solve the monodromy and stability problems (except when the principal term of the Dulac development of the return map is the identity) for degenerate singular points of a generic family of vector fields. It is known, that for the most cases, the stability of degenerate monodromic points can be determined integrating the first order variational equations, associated to the edges of the the first polar blow-up polycycle. We are motivated by the fact that this family contains cases, such that the contribution to the stability of the point, «hidden» in the singular points of the first polar blow-up polycycle, compensates the contribution given by the first order variational equations associated to the edges of the polycycle. This property is used to generate a bifurcation of a limit cycle.

Linearizability and Complex Isochronous Saddles

Colin Christopher **Plymouth University, England**
Pavao Mardesic **Université de Bourgogne, France**
Christiane Rousseau **Université de Montreal, Canada**

Abstract: This is a part of a joint work with C. Christopher and C. Rousseau. We define the notion of isochronicity for integrable saddles and prove that an integrable saddle is

isochronous if and only if it is linearizable. We also generalize this result to normalizable systems.

Some Remarks about the Integration of Polynomial Planar Vector Fields

Jean Moulin-Ollagnier **Laboratoire GAGE. École Polytechnique, France**

Abstract: The aim of this talk is to present some tools, especially but not only from enumerative and algebraic geometry, that are involved in the search of first integrals of polynomial planar vector fields. As an illustration, we describe how these tools have been useful in our recent characterization of all cases of Liouvillian integration of the homogeneous three-dimensional Lotka-Volterra system.

On the Hypersurface Solutions to a Pfaff Equation

Marcel Nicolau **Universitat Autònoma de Barcelona, Spain**

Abstract: We discuss some generalizations of Jouanoulou's theorem on the finiteness of hypersurface solutions to a Pfaff equation without first integral.

Degeneracy in Periodic Equations

Rafael Ortega **Universidad de Granada, Spain**

Abstract: Consider the equation $$x»+g(t,x)=s$$ or $$x'+g(t,x)=s,$$ where g is periodic in time and s is a real parameter. This equation will be called degenerate if it has no periodic solution for any $sneq 0$. In the theory of forced oscillations it is useful to characterize the class of degenerate equations.par This problem could have some similaritiy with the problem of isochronous centers of autonomous equations.

Integrable Quadratic Vector Fields with Generic Algebraic Solutions

Jaume Llibre **Universitat Autònoma de Barcelona**
Jesús S. Pérez del Río **Universidad de Oviedo, Spain**
J. Ángel Rodríguez **Universidad de Oviedo, Spain**

Abstract: Darboux in 1878 was the first in showing the fascinating relationships between the integrability (a topological phenomena) and the existence of invariant algebraic curves. Several authors have obtained improvements to Darboux theory of integrability. Recently, Christopher and Zholadek have proved the next result: If X is a polynomial vector field of degree n that has algebraic solutions such that the sum of their degrees is n+1 and they satisfy some conditions of genericity, then X is integrable. Moreover, X can be expressed through these solutions and their associated Hamiltonian vector fields. In this work we consider n=2 and we study the vector fields that has an invariant curve of degree 3 such that their irreducible factors verify the assumptions of Christopher and Zholadek. We do their topological classification in the Poincar'e's compactificacion and we obtain 46 equivalence classes.

On the Double-Zero Unfolding in Symmetric Piecewise Linear Systems

Enrique Ponce **Universidad de Sevilla, Spain**

Abstract: We are interested in the dynamic bifurcations, giving rise to limit cycles, that appear organized by the double-zero bifurcation point in symmetric piecewise linear systems. One of the lines emanating from this point corresponds with vertical Hopf points, and the information about limit cycle amplitude and period evolution is well known. However, similar results concerning a degenerate pitchfork bifurcation which also involves the birth of a symmetric pair of limit cycles are not available. Some results concerning this other dynamic bifurcation will be shown.

Bifurcation of Limit Cycles from Hamiltonian Systems

Tomeu Coll **Universitat de les Illes Balears, Spain**
Armengol Gasull **Universitat Autònoma de Barcelona, Spain**
Rafael Prohens **Universitat de les Illes Balears, Spain**

Abstract: We study the number of limit cycles that bifurcate from the periodic orbits of a center in two cases of planar Hamiltonian systems. We do this by perturbing the Hamiltonian systems inside either the class of all polynomial systems or in a family of rational functions, and studying the number of isolated zeroes of the Abelian integral. As a consequenc, we provide lower bounds for the Hilbert numbers in terms of the degree of the system.

On the Polynomial Vector Field of Degree n with n-1 Algebraic Limit Cycles

Rafael Ramírez **Universitat Rovira i Virgili, Spain**
Natalia Sadovskaia **Universitat Politècnica de Catalunya, Spain**

Abstract: We construct a polynomial vector field of degree n with $n-1$ invariant circumferences. The 16th Hilbert problem for algebraic limit cycles is study.

Invariant Curves and Topological Invariants for Real Plane Analytic Vector Fields

Jean Jacques Risler **Université Pierre et Marie Curie, Paris VI, France**

Abstract: Let Z be a germ of a singular real analytic vector field at $O in {bf R}^2$. We give conditions on the multiplicity and the Milnor number of Z which imply that the foliation defined by Z has a characteristic orbit, or an analytic invariant curve with the hypothesis that Z is a «Real Generalized Curve». Then it is proved that for a non-dicritical «Real Generalized Curve», the multiplicity mod 2 is invariant under bilipshitz homeomorphisms preserving foliations.

Not Notified

J. Ángel Rodríguez **Universidad de Oviedo, Spain**

Abstract: Not Notified

Reduction of Several to one Parameter in Analytic Families of Planar Vector Fields

Robert Roussarie **Université de Bourgogne, France**

Abstract: The computation of the number of limit cycles which appear in an analytic unfolding of planar vector fields is related to the decomposition of the displacement function of this unfolding in an ideal of functions in the parameter space, called the Ideal of Bautin. On the other hand, the asymptotic of the displacement function, for 1-parameter unfoldings of hamiltonian vector fields is given by Melnikov functions which are defined as the coefficients of Taylor expansion in the parameter. It is interesting to compare these two notions and to study if the general estimations of the number of limit cycles in terms of the Bautin ideal could be reduced to the computations of Melnikov functions for some 1-parameter subfamilies.

Analytic Normal Form for Saddle-Nodes and Finite Cyclicity of Graphics

Christiane Rousseau **Université de Montreal, Canada**

Abstract: The lecture will present a refinement of the transformation to smooth normal form for an analytic family of vector fields in the neighborhood of a saddle-node. This allows to prove the finite cyclicity of families of graphics («ensembles») occuring inside analytic families of vector fields, for instance the «lips» in generic conditions. It is essential for the proof of the finite cyclicity of an hp-graphic through a nilpotent singular point of elliptic type. It allows substantial improvement in the program to prove the finiteness part of Hilbert's 16th problem for quadratic vector fields (by showing the finite cyclicity of 121 graphics).

On the Number of Large Amplitude Limit Cycles of Second Order Polynomial ODE's

Marco Sabatini **Universitá de Trento, Italy**

Abstract: In this paper we study the number of limit cycles of polynomial Li'enard equation $$ x» + f(x)x' + g(x) = 0 $$ that bifurcate from infinity when a suitable perturbation is introduced. We show that for every positive integer n there exists a polynomial Li'enard equation of degree $2n+1$ having n limit cycles of large amplitude. Similar results are proved for other classes of {rm II} order ODE's.

Balanced Coordinates for Spiraling Vector Fields

Fernando Sanz **Universidad de Valladolid, Spain**

Abstract: Let X be an analytic vector field in $(R^3,0)$ with a {em twister axis/} $Gamma$ in the sense of [C-M-S]. That is, $Gamma$ is an invariant analytic curve for which there is a neighbourhood composed of integral curves $gamma$ of X that {em spiral/} asymptotically around the axis $Gamma$ and have flat contact with it. Suppose that $Gamma$ is a smooth curve and take some coordinates (x,y,z) such that $Gamma$ is the z-axis. The spiraling behaviour of such an integral curve $tmapsto gamma(t)=(x(t),y(t),z(t))$ is given by the fact that its angle $theta(t)=arctan{y(t)/x(t)}$ is divergent while the distance $r(t)=sqrt{x^2(t)+y^2(t)}$ from the axis goes to zero. We say that the coordinates (x,y,z) are {em balanced/} if the

vector field X is transversal to the foliations ${y/x=constant}$ and ${x^2+y^2=constant}$. This means that the angle and the distance from $Gamma$ is monotone and have a {em uniform/} behaviour for any integral curve $gamma$. We show with some examples that this uniformity depends on the coordinates. We prove that there always exist some balanced coordinates in the case of {em elementary singularity/} (non nilpotent linear part) and $Gamma$ not contained in the singular locus of X. Also, with this last condition and in the case of nilpotent singularity, we show the existence of coordinates for which the angle is monotone. These are obtained as coordinates giving {em maximal contact/} for the reduction of the singular point. The results can be viewed as generalizations of the same results in dimension two which we study first as a motivation.par vspace{5pt} [C-M-S] Cano, F.; Moussu, R.; Sanz, F.: «Oscillation, Spiralement, Tourbillonnement». Comm. Math. Helv., 75 (2000), 284-318.

Concepts for the Classification Problems of Planar Quadratic Systems

Dana Schlomiuk **Université de Montreal, Canada**

Abstract: In this lecture we first take a look at the literature on quadratic systems in the direction of the classification problems. We next proceed by introducing several geometric concepts helpful for encoding and organizing the massive information encountered in classification problems. We show how these concepts link the geometrical study of the vector fields to the algebraic invariant theory of the systems and clear the way for specific analytic studies, theoretical and numerical. We illustrate this by specific examples and results.

Geometry of Quadratic Cycles

Douglas S. Shafer **University of North Carolina at Charlotte, USA**

Abstract: We investigate the geometry and affine geometry of periodic trajectories that occur in the phase portraits of quadratic systems of differential equations.

Umbilic and Tangential Singularities on Lines od Curvature Configurations in Surfaces with Boundary

Jorge Sotomayor **Universidade de Sao Paulo, Brazil**

Abstract: A relationship between the umbilic points that appear near the border of a surface which is approximated by the border of a tubular neighbourhood is studied. It is stablished that umbilics bifurcate from points of tangency to the border.

Mathematical Works of M.N. Lagutinskii (1871-1915)

Jean Marie Strelcyn **Université de Rouen and LAGA, Institut Galiée, Université Paris 13, France**

Abstract: The forgotten Russian mathematician M.N. Lagutinskii (1871-1915) (see [1] for his biography) was a successor of G. Darboux in what concern the algebraic Darboux method of

search of first integrals in finite terms of polynomial systems of ODE. The aim of my talk is to present some mathematics results obtained by M.N. Lagutinskii in this area.

REFERENCES [1] V.A. Dobrovol'skii, N.V. Lokot', J.-M. Strelcyn – Mikhail Nikolaevich Lagutinskii (1871-1915): Un math\'ematicie m\'econu, Historia Mathematica, Vol.25(1988), 245-264.

Isochronous Centers and Real Invariant Straight Lines in Cubic Planar Systems

Javier Chavarriga **Universitat de Lleida, Spain**
Eduardo Sáez, Iván Szántó **Universidad de Santa María, Chile**

Abstract: We prove the existence of one-parameters families of cubic isochronous centers with three real invariant stright lines.

Reversible Unfolding of Planar Degenerate Cusps

Ronaldo García **Universidade Federal de Goiaz, Brazil**
Marco Antonio Teixeira **Universidad de Campinas, Brazil**

Abstract: In this talk we discuss the bifurcation diagram of the singularity of the vector field $X(x,y)=(y , 2x^5 + 2x^3 y))$ in the class of reversible vector fields. The three parameter unfolding of such system is also established.

Limit Cycles, Poincaré Maps and First Integrals for Piecewise Linear Systems

Jaume Llibre, E. Núñez, Antonio E. Teruel **Universitat Autònoma de Barcelona, Spain**

Abstract: In the qualitative theory of differential equations, research on limit cycles is an interesting and difficult part. In order to determine whether there existed a limit cycle for a given differential equation and to study the properties of limit cycles, Poincar\'e introduced the successor function, what is actually know as Poincar\'e map. Using Poincar\'e maps we present some relevant results about the number of limit cycle and their properties for the family of fundamental systems. Fundamental systems are a particular case of piecewise linear system. They are three pieces continuous linear systems, and symmetric with respect to the origin. The same results as before can be achieved using first integrals of fundamental systems.

Existence of Periodic Orbits via Fuller Index and Averaging Method

Massimo Villarini **Universitá di Modena e Reggio Emilia, Italy**

Abstract: The problem of existence of periodic orbits for perturbations of vector fields generating a fibration by circles on a manifold is considered: relevant examples are coupled oscillators having the same frequencies, or the geodesic flow on spheres.An existence theorem of Seifert and Fuller is generalized via a connection with the averaging method of one frequency systems.Some examples will be discussed.

Local First Integrals of Differential Systems

Weigu Li **Peking University, China**
Jaume Llibre **Universitat Autònoma de Barcelona, Spain**
Xiang Zhang **Nanjing Normal University, China**

Abstract: In this paper using theory of linear operators and normal forms we generalize a result of Poincar\'e about the non-existence of local first integrals for systems of differential equations in a neighbourhood of a singular point. As an application of the generalized result, and under more weak conditions we obtain a result of Furta about local first integrals of semi-quasihomogeneous systems. Moreover, for periodic differential systems we give definitions of their first integrals, and generalize the previous results about systems of differential equations to periodic differential systems in a neighbourhood of a constant solution.

Planar Periodic Systems Without Periodic Solutions

Henryk Zoladek **University of Warsaw, Poland**

Abstract: We construct examples of planar systems of the form $dz/dt= z^n+p_{n-1}(e^{it})z^{n-1}+\ldots + p_0(e^{it})$ (where p_j are polynomials) which do not have periodic solutions. For the Riccati system $dz/dt=z^2+re^{it}$ we express the values r_j of the parameter for which there is no periodic solution by means of zeroes of the Bessel function J_0.